Control and Adaptation of Spatio-temporal Patterns

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We apply a recently introduced cognitive system for dynamics recognition to a two-dimensional array of coupled oscillators. The cognitive system allows for both the control and the adaptation of spatio-temporal patterns of that array of oscillators. One array that shows Turing-patterns in a self-organizational manner is viewed as an externally presented dynamics (stimulus) which is mapped onto a mirror dynamics, whereby the latter is capable to simulate (simulus). Two of the parameters of the stimulus are thereby regarded to be unknown and have to be estimated by the cognitive system. The cognitive system itself consists of dynamical modules that are stimulated by the external dynamics in the sense of Pyragas' external force control mechanism and thereby yield measures of how good they match the stimulus. These measures are used as weights to construct the simulus. The adaptation process is performed "on the fly", i. e., without the storage of data. The proposed cognitive system, therefore, is a prominent candidate for the construction of a control device for a permanent real time observation of an external dynamical system in order to interfere instantaneously when necessary.

Key words: Adaptive Systems; Cognitive Systems; Pattern Formation; Brain Dynamics; Force Control.

1. Introduction

Undoubtedly, the brains of higher developed species have a simulation capability which makes them act as autonomous optimizers [1 - 4]. Moreover, in the case of humans it has been plausibly argued that the brain is a hermeneutic device [5, 6] which is obviously linked to the simulation capability [7 - 10]. Guided by this important simulation aspect of brain dynamics we present a system which mimics such a behavior. The basic principle of the system has been described in previous publications where we convincingly adapted a low-dimensional dynamics to a scalar (lumped variable) time-series. In the present paper we show that it is also possible to control spatio-temporal Turing-patterns in two spatial dimensions and to adapt to them.

The scheme of our system is depicted in Figure 1. To the left, we have an external dynamics (the stimulus) which is "perceived" by the system. To the right, we see the representation of the external dynamics as a "mirror". This dynamics is the simulative part of the system (simulus). The simulus undergoes an adaptation procedure that is described below.

The kernel of the system is a pool of dynamical modules, each of which is controlled by the external dynamics. The pool of internal systems is shown in the middle part of Fig. 1 as a stack of six modules, where six is an arbitrary number. The strength of the control term, which is explained in detail in the following section, serves as a measure of the "fitness" of the corresponding module. The parameters of the modules are used in a superpositional manner weighted by their "fitness" to build up the parameter of the simulus. In the scheme depicted in Fig. 1 the modules D_3 and D_4, for example, fit best to the stimulus and, therefore, contribute with a larger weight (bold arrows) to the construction of the simulus. D_1 and D_6, for example, fit worse and contribute little (dotted arrows) to build up the simulus. The simulus is itself forced by the stimulus and can in so far also be seen as a module. However, in contrast to

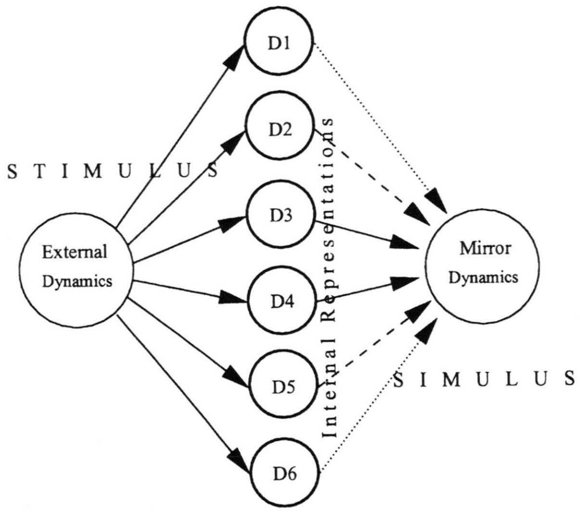


Fig. 1. Scheme of the adaptive system. An external dynamics is presented to the cognitive system as a stimulus. The cognitive system is substantially build up by a pool of internal dynamical modules. In the paper in hand the modules' dynamics are restricted to the same dynamical type as the stimulus and are defined through a set of parameter values. The stimulations of the modules are performed by means of a force control method which allow for the adaptation of a mirror dynamics to the stimulus. The mirror dynamics can be called simulus since it is capable to simulate the stimulus. Please see the text for a detailed description.

the modules D_1 through D_6, the simulus is capable to adapt to the stimulus which is why the simulus is treated as a separate unit.

We emphasize that the adaptation is done "on the fly". We do not have to record a long time series of the external dynamics and analyze it by means of established time series analyses which are far too slow for the purpose of an instantaneous response. In addition, common methods rely on a stationarity of the external dynamics in the long run what we do not need to assume here.

In the following section we present the mechanism for the "perception" which is performed on the basis of the so called Pyragas external force control [11].

2. Brief Recapitulation of the Pyragas Control Method

The external force control method introduced by Pyragas [11] in its original form deals with the stabilization of unstable periodic orbits in nonlinear dynamic systems in the chaotic regime. Control is achieved by adding a "control term" (which is proportional to the difference between a variable of the system and the corresponding projection of the unstable periodic orbit to be stabilized) to the corresponding differential equation of the system. This method is able to stabilize unstable periodic orbits of the system with an – in the long run – vanishing control term.

In the following we deviate slightly from this original application by using Pyragas' control method for synchronization of two dynamical systems and refraining from being able to stabilize with an almost vanishing control term. Assume $\boldsymbol{x}$ and $\boldsymbol{x}'$ to be the states of two dynamical systems of the same dimension n and the same dynamics $\boldsymbol{f}$, which are given by the differential equations

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x};\boldsymbol{\beta}), & \boldsymbol{\beta} &= (\beta_1,\beta_2,\ldots,\beta_m), \\ \dot{\boldsymbol{x}}' &= \boldsymbol{f}(\boldsymbol{x}';\boldsymbol{\beta}'), & \boldsymbol{\beta}' &= (\beta_1',\beta_2',\ldots,\beta_m'), \end{aligned} \tag{1}$$

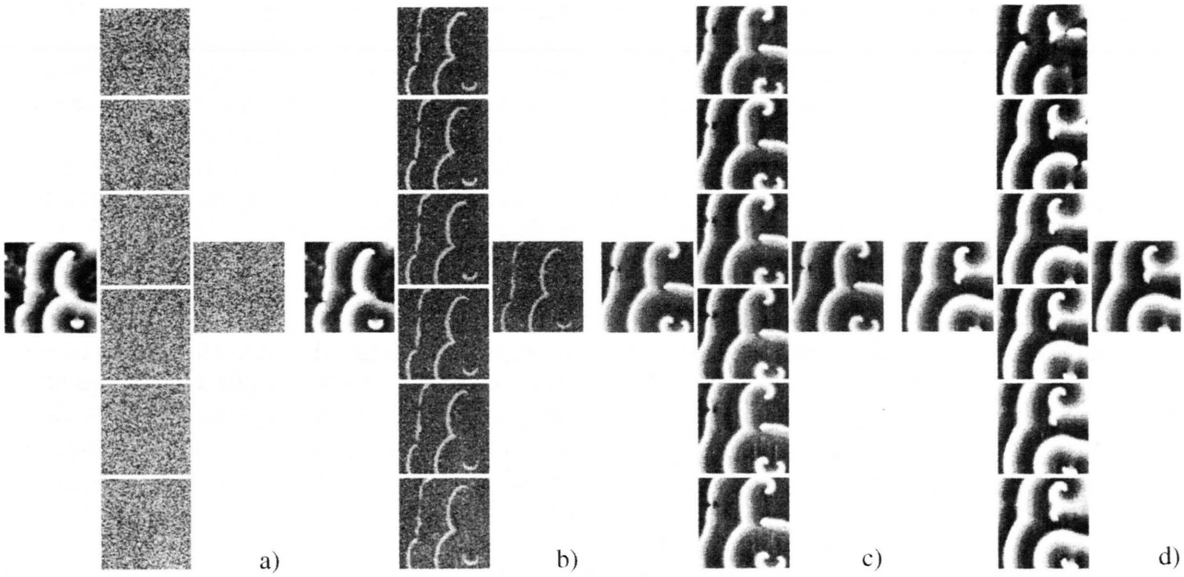
where $\boldsymbol{\beta}$ and $\boldsymbol{\beta}'$ are sets of fixed parameters. If now the difference of at least one (appropriate) pair of corresponding variables (say the first) multiplied by a suitably chosen factor K is added to the unprimed system,

$$\begin{aligned} \dot{x}_1 &= f_1(x_1,x_2,\ldots,x_n;\beta) + K(x_1' - x_1) \\ \dot{x}_2 &= f_2(x_1,x_2,\ldots,x_n;\beta) \\ &\vdots \\ \dot{x}_n &= f_n(x_1,x_2,\ldots,x_n;\beta), \end{aligned} \tag{2}$$

this unprimed system will be forced to the dynamics of the primed controlling system, at least if the difference of the dynamics is not too extreme [12]. The value of the control term $K(x_1' - x_1)$ may be used as a measure for the quality of the control. As in the original application of Pyragas' method, this control term will be negligible in the long term if the difference of the system parameters is relatively small.

3. The Stimulus: An Array of Coupled Oscillators

We now describe the system which we treat as an externally given dynamics that is "perceived" by our cognitive system. This stimulus is a two-dimensional array of 100×100 diffusively coupled oscillators that shows Turing-patterns in form of spiral waves that emerge without an external excitation, however, sustain in a dynamical movement (cf. Fig. 2).



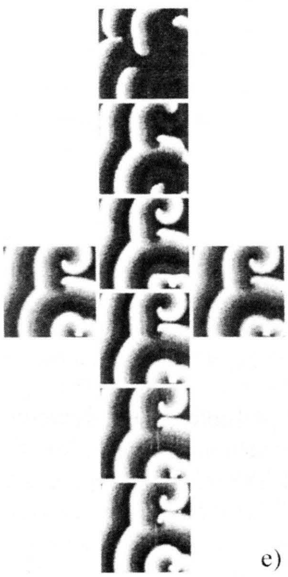


Fig. 2. Five (a through e) screenshots of greyscale representations of the adaptive system. The arrangement of the modules of each cognitive systems fully corresponds to the scheme of Figure 1. A detailed explanation is given in the text.

Specifically, the stimulus that we use in the sequel reads

$$\begin{aligned} \dot{x}^{(\mathrm{E})}_{i,j} &= a + e x^{(\mathrm{E})}_{i,j} y^{(E)}_{i,j} - \frac{c x^{(\mathrm{E})}_{i,j}}{d + x^{(\mathrm{E})}_{i,j}} + D_x \, \mathrm{Diff}^{x}_{i,j}, \\ \dot{y}^{(\mathrm{E})}_{i,j} &= b - x^{(\mathrm{E})}_{i,j} y^{(E)}_{i,j} + D_y \, \mathrm{Diff}^{y}_{i,j} \end{aligned} \qquad (3)$$

with $i, j = 1, \ldots, 100$. The parameters of the stimulus are kept fixed throughout the paper and are given by $a = 0.0644$, $b = 0.12$, $c = 0.575$, $d = 0.1$, $e = 1.15$, $D_x = 0.1$, $D_y = 0.02$. The parameters a and e are regarded as being unknown and have to be estimated. The superscript E stands for "external" to distinguish the system from the later-used internal systems.

We use zero-flux boundary conditions for the diffusion Diff^{ξ}:

$$\mathrm{Diff}^{\xi}_{i,j} = \begin{cases} \xi_{i+1,j} + \xi_{i-1,j} + \xi_{i,j+1} + \xi_{i,j-1} - 4\xi_{i,j}, & 1 < i < 100;\ 1 < j < 100, \\ \xi_{i+1,j} + \xi_{i,j+1} + \xi_{i,j-1} - 3\xi_{i,j}, & i = 1;\ 1 < j < 100, \\ \xi_{i-1,j} + \xi_{i,j+1} + \xi_{i,j-1} - 3\xi_{i,j}, & i = 100;\ 1 < j < 100, \\ \xi_{i+1,j} + \xi_{i-1,j} + \xi_{i,j+1} - 3\xi_{i,j}, & 1 < i < 100;\ j = 1, \\ \xi_{i+1,j} + \xi_{i-1,j} + \xi_{i,j-1} - 3\xi_{i,j}, & 1 < i < 100;\ j = 100, \\ \xi_{i+1,j} + \xi_{i,j+1} - 2\xi_{i,j}, & i = 1;\ j = 1, \\ \xi_{i-1,j} + \xi_{i,j-1} - 2\xi_{i,j}, & i = 100;\ j = 100, \\ \xi_{i-1,j} + \xi_{i,j+1} - 2\xi_{i,j}, & i = 100;\ j = 1, \\ \xi_{i+1,j} + \xi_{i,j-1} - 2\xi_{i,j}, & i = 1;\ j = 100, \end{cases}$$

$$\xi \in \{x, y\}. \qquad (4)$$

The array of oscillators given by (3) and (4) is a variant of the one that has been used to describe chemical spatio-temporal pattern formations in diffusively coupled cells [13 - 15]. It is dynamically similar to continuous neuronal networks of the Hodgkin-Huxley type [14].

In previous publications [8 - 10] we have shown that a scalar external signal (lumped sum variable) can be recognized and simulated in a convincing fashion in low-dimensional systems even if the signal is chaotic. The present application of the adaptive cognitive system to a high-dimensional dynamical system is an important step towards the recognition of and the adaptation to moving images.

4. The Internal Dynamics Modules

We assume that the type of dynamics of the stimulus given by (3) and (4) is known. However, the values of the two parameters a and e are regarded to be unknown. The estimation of these two parameters is the task of the adaptation procedure described in the following. In doing so we introduce a pool of internally given dynamical modules that more or less accurately mimic the external system. Each of these modules is forced by the external array by means of (2) which explicitly leads to

$$\begin{aligned} \dot{x}_{i,j}^{(k)} &= a^{(k)} + e^{(k)} x_{i,j}^{(k)} y_{i,j}^{(k)} - \frac{c x_{i,j}^{(k)}}{d + x_{i,j}^{(k)}} \\ &\quad + D_x \mathrm{Diff}_{i,j}^{x^{(k)}} + K(x_{i,j}^{(\mathrm{E})} - x_{i,j}^{(k)}) \\ \dot{y}_{i,j}^{(k)} &= b - x_{i,j}^{(k)} y_{i,j}^{(k)} + D_y \mathrm{Diff}_{i,j}^{y^{(k)}}, \end{aligned} \tag{5}$$

where the coupling constant K is chosen to be 0.3 for all modules throughout the paper. This "perception" mechanism is schematically depicted in Figure 1. We use six modules in the following where the superscript ($(k) = 1, \ldots, 6$) in (5) indicates the number of the module. The modules span a grid with respect to the parameter values of $a^{(k)}$ and $e^{(k)}$, respectively. In an alternating fashion we choose six different values for the $a^{(k)}$ and equal values for the $e^{(k)}$, respectively, during a certain time interval followed by an interval in which we choose six different values for $e^{(k)}$ and equal ones for $a^{(k)}$, respectively. The precise procedure is explained in the subsequent section.

We mention in passing that alternatively an internal pool of 36 modules using all the 6×6 parameter combinations of $a^{(k)}$ and $e^{(k)}$ could be used to perform the adaption of both parameters simultaneously. However, this would enlarge the dimension of the cognitive system. It depends on the context whether this or rather the procedure used in the following is more adequate. Since we focus here on the basic mechanism of pattern adaptation, we postpone further investigations to a forthcoming paper.

If the parameter value of the k^{th} module is in the vicinity of the true parameter value of either a or e, depending on which parameter is currently being adapted, the cumulative control strength $C^{(k)} = \sum_{i=1}^{100} \sum_{j=1}^{100} |x_{i,j}^{(\mathrm{E})} - x_{i,j}^{(k)}|$ is expected to be small whereas the corresponding control strength of a module whose parameter value is far from the true one should be large. The idea now is to use the inverse control strengths as weights for the contribution of the corresponding parameter values to a linear combination in order to construct a new parameter, namely that one of the mirror system that gradually builds up a simulative counterpart of the stimulus.

Eventually, the above reasoning motivates the formulation of a differential equation for the adaptation of the parameter values in the following way:

$$\dot{\alpha}^{(\mathrm{M})} = \frac{\sum_{k=1}^{6} \frac{1}{C^{(k)}(t)} \left(\alpha^{(k)}(t) - \alpha^{(\mathrm{M})}(t)\right)}{\frac{s}{C^{(\mathrm{M})}(t)} + \sum_{k=1}^{6} \frac{1}{C^{(k)}(t)}}. \tag{6}$$

In this equation α stands for either a or e, respectively, depending on the actual time interval of the adaptation procedure. M indicates the mirror system that also obeys (5) (with (k) replaced by (M)), however, with time-dependent parameters $a^{(\mathrm{M})}$ and $e^{(\mathrm{M})}$, respectively, according to (6). The mirror system (simulus) itself undergoes controlling and participates at the linear combination. In other words, the mirror parameters $\alpha^{(\mathrm{M})}$ are able to confirm their own values if they lie in the vicinity of the true one. Additionally, (6) contains a smoothing parameter s which serves as a self-affirmation parameter. This parameter avoids too rapid changes in the value of α and thus reduces fluctuations caused by desynchronization of the stimulus with the simulus. The process of adaptation of the parameter values is described in more detail in [10].

5. Adaptation Results

All parts of the system, the stimulus, the internal modules, the simulus as well as the parameters that are

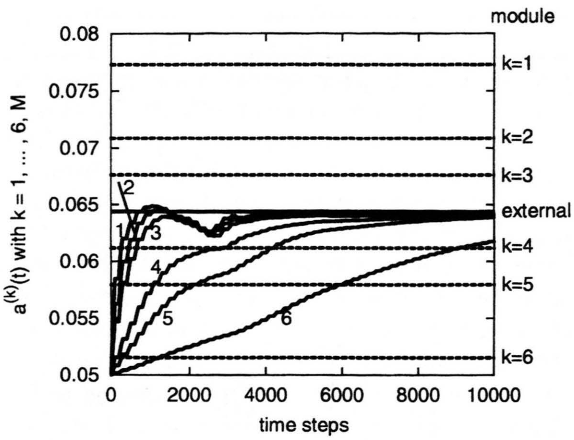


Fig. 3. Time courses of the adapted mirror parameter $a^{(M)}$ for six different values of the smoothing parameter s together with the parameter values $a^{(k)}(k = 1, \ldots, 6)$ of the six internal modules. The modules are labeled at the right with $k = 1$ through $k = 6$. The constant value of the external parameter a that has to be estimated is also shown and labeled with "external". The six graphs labeled 1 throught 6 correspond to the six runs using $s = 1, 5, 10, 50, 100, 500$.

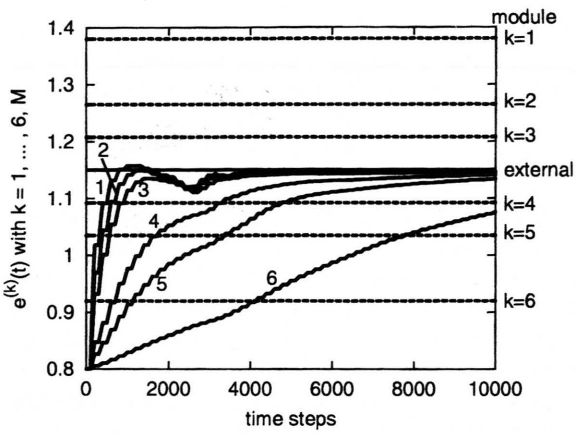


Fig. 4. Time courses of the adapted mirror parameter $e^{(M)}$ for six different values of the smoothing parameters s together with the parameter values $e^{(k)}(k = 1, \ldots, 6)$ of the six modules. The graphs have been labeled fully analogously to Fig. 3.

adapted, are integrated using a time step of $h = 0.01$. Figure 2 shows five screenshots of the complete system. Thereby the array of oscillators is depicted as a greyscale encoding of the values of the x-variables of the oscillators. The arrangement of the individual elements of the cognitive system in Fig. 2 fully corresponds to the scheme of Figure 1. An early screenshot of the system can be seen in Figure 2a. The stimulus (left part) shows spiral patterns from the beginning. The transient has been skipped in a very long run before the adaptation started. The six internal modules (arranged in the middle as a stack) have been initialized in random (equi-distributed) states for each oscillator, which leads to a diffuse pattern as can be seen in Figure 2a. Some of the internal modules are capable to form patterns after a while without stimulation (which is not shown), others remain in a diffuse state.

The precise adaptation procedure works as follows. Within the first 100 iteration steps of the integration the six modules are defined through the six parameter values

$$a^{(1)} = 0.0773, \quad a^{(2)} = 0.0708, \quad a^{(3)} = 0.0676,$$
$$a^{(4)} = 0.0612, \quad a^{(5)} = 0.0580, \quad a^{(6)} = 0.0515, \qquad (7)$$

whereas all $e^{(k)}$ have been set equal to 0.80 which is also the initial value of the parameter $e^{(M)}$ of the mirror system. The initial value of $a^{(M)}$ is 0.05.

During the second interval of 100 more time steps length, all the $a^{(k)}$ have been set to the final value of the mirror parameter $a^{(M)}$ reached at the end of the first adaptation interval, whereas the $e^{(k)}$ have now been set to the six values

$$e^{(1)} = 1.380, \quad e^{(2)} = 1.265, \quad e^{(3)} = 1.208,$$
$$e^{(4)} = 1.093, \quad e^{(5)} = 1.035, \quad e^{(6)} = 0.920. \qquad (8)$$

During the third interval all the $e^{(k)}$ have been set to the final value of $e^{(M)}$ reached at the end of the second interval, and the $a^{(k)}$ have been set again to the six values given by (7), and so on in an alternating fashion.

The control processes given by (5) quickly force the modules to synchronize with the stimulus, which results in a mimicking of the spiral pattern. Figure 2b is a screenshot made after about 200 time steps. One sees that the spiral patterns emerge gradually. After about 500 time steps the external pattern and the internal ones in the modules can hardly be distinguished (which is not shown in the figure). Figure 2c shows the system after roughly 5000 time steps. All modules and the external array are fully synchronized.

Since the parameter values of the mirror system, i.e. $a^{(M)}$ and $e^{(M)}$, quickly adapt to the parameter values of the stimulus, the mirror system is capable to simulate in a rather accurate way for a long time after switching off the control – which justifies the name "simulus". The time courses of the parameter adaptations of $a^{(M)}$ and $e^{(M)}$, respectively, are shown in Figs. 3 and 4, respectively, for six different values of the smoothing parameter s, namely $s = 1, 5, 10, 50, 100, 500$. One sees a quick adaptation for $s = 1$, however with a transient oscillation in the beginning and a small persisting fluctuation around the final value. The larger the value of the smoothing parameter the smaller are the remaining fluctuations, however, on the cost of the speed of adaptation.

If the force control is switched off, the internal modules quickly desynchronize with the stimulus and the patterns either vanish or evolve in a way fully independent from the pattern of the stimulus. Not so in the case of the simulus. Figures 2d and 2e show two screenshots after the decoupling of the simulus as well as the internal modules from the stimulus. Figure 2d is a screenshot made after 2000 time steps after the decoupling. One sees that the six modules begin to desynchronize with the stimulus. The uppermost module, whose parameter value differs most from that of the stimulus, is desynchronized most. After 4000 time steps (Fig. 2e) some of the modules are fully desynchronized, whereas the simulus is still in good synchronization with the stimulus.

6. Conclusions

Metaphorically speaking, we introduced a pool of simulated "parallel worlds" that are compared with what happens in the real world. Thereby, the "fitness of matching" is used to create a "mirror" world as an internal representation of the real one. The used mechanism is a variant of the Pyragas' force control which is an especially adequate method because of its first-principle nature [16, 17]. In [16] it has been plausibly argued that the Pyragas method is a prominent candidate for control processes in the brain. In [17] it has been shown that the Pyragas method can be led back to a fundamental diffusive process on a molecular level. Thus, the proposed cognitive system obeys a strikingly simple and most fundamental mechanism.

The performance of the cognitive system depends crucially on the chosen number of modules, six in our case. With respect to the computation time consumption one may wish to have as few as possible. Due to the weighted linear superposition as given by (6) it is clear that the parameter value of the external system which has to be estimated has to lie somewhere inbetween the parameter values of the modules. Additionally, the initial value of the mirror parameter has to be chosen inbetween these limiting values. Therefore, the upper and the lower border values have to be chosen with a large enough spacing so that the unknown stimulus parameter value is covered with certainty. A large spacing, on the other hand, leads to a reduction of precision of the estimation. This can be tackled by using more modules that reduce the spacing and, therefore, increase the precicion of the adaptation. In the described example a number of six modules turned out to be a good choice. We hope that further developments of the cognitive system will allow for an autonomous optimization of the number of modules by the system itself.

In principle, the mirror world – which we call simulus – can be fed back to the pool by removing such internal worlds that always or extremely mismatch with the real world and may, therefore, be regarded as relatively useless. Thus, the set of internal dynamics modules undergoes a continuous reconfiguration. This means that we can combine self-modifying features with memory, which may serve to even introduce creativity aspects. To memorize modules for later use relativates the usage of a large number of modules as discussed above. If the variability of the presented stimuli is large it may turn out as an advantage to have a larger number of modules even at the cost of computation time.

The proposed adaptive system has some similarity with the Bayesian learning [18]. The Bayesian inference is used to find an *a posteriori* probability for the validity of a given hypothesis after a test or an experiment has been carried out. Such an update of an *a priori* probability, however, only yields probabilities for the predefined hypotheses. It cannot create new hypotheses itself. The proposed adaptive system, however, endowed with self-modifying features via internal control (cf. [16, 19]) and mutations, very likely can overcome this restriction. Self-modifying features may also allow for overcoming the restriction to mere parameter adaptations as discussed in this paper. We are very confident that it will be soon possible to implement also features into the adaptive system that allow for the adaptation of the whole dynamical type and not only of the parameter values of otherwise fixed dynamical types. It seems to be justified then to speak of an "hermeneutic engine" in accordance with Erdi [5] and Kaneko and Tsuda [6, 20].

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